Erratum: Transition records of stationary Markov chains [Phys. Rev. E 74, 040103(R) (2006)]

Jan Naudts and Erik Van der Straeten
(Received 8 June 2007; published 29 June 2007)

DOI: 10.1103/PhysRevE.75.069902 PACS number(s): 05.70.Ln, 05.40.−a, 99.10.Cd

In this paper, in the section on the fluctuation theorem, an error was made by intuitively assuming that the probability distribution $p(x)$ is stationary. The error is corrected by including the ratio of the probabilities of initial and final points of the path to the definition of the entropy production variable $W_x(k)$. In a per path notation it becomes

$$W(\gamma) = \ln \frac{p(\gamma_i)w(\gamma)}{p(\gamma_f)w(\bar{\gamma})}. \tag{24}$$

This variable still is constant on each of the classes $C(x,k)$. However, its value has an additional contribution $\ln[p(\gamma_i)] - \ln[p(\gamma_f)]$. The fluctuation theorem then becomes as follows.

Theorem 2. In any Markov chain with finite state space the entropy production variable satisfies

$$\frac{\mathrm{Prob}(W=K)}{\mathrm{Prob}(W=-K)} = e^K. \tag{26}$$

Proof. One has

$$\begin{aligned}\mathrm{Prob}(W=K) &\equiv \sum_\gamma p(\gamma_i)w(\gamma)\delta_{\{W(\gamma),K\}} = e^K \sum_\gamma p(\gamma_f)w(\bar{\gamma})\delta_{\{W(\gamma),K\}} = e^K \sum_\gamma p(\gamma_i)w(\gamma)\delta_{\{W(\bar{\gamma}),K\}} \\ &= e^K \sum_\gamma p(\gamma_i)w(\gamma)\delta_{\{W(\gamma),-K\}} = e^K \mathrm{Prob}(W=-K).\end{aligned} \tag{27}$$

This ends the proof. ■

The average value of the entropy production variable satisfies

$$\langle W\rangle = \frac{1}{2}\sum_\gamma [p(\gamma_i)w(\gamma) - p(\gamma_f)w(\bar{\gamma})]\ln \frac{p(\gamma_i)w(\gamma)}{p(\gamma_f)w(\bar{\gamma})}.$$

It is immediately clear that it is positive on a per term basis. However, the positivity also follows from the fluctuation theorem, with the argument given in the paper

$$\langle W\rangle = \sum_K K\,\mathrm{Prob}(W=K) = \frac{1}{2}\sum_K \mathrm{Prob}(W=K)K(1-e^{-K}) \geqslant 0. \tag{29}$$

The correct relation between the entropy production $\langle W\rangle$ and the dynamical entropies is

$$\bar{S}_\theta^{(n)} - S_\theta^{(n)} = \langle \bar{\Psi}_\theta\rangle - \langle \Psi_\theta\rangle = \langle W\rangle - \sum_\gamma p(\gamma_i)w(\gamma)\ln \frac{p(\gamma_i)}{p(\gamma_f)}. \tag{28}$$

If p is stationary then the latter term vanishes and Eq. (28) of the paper is recovered.

